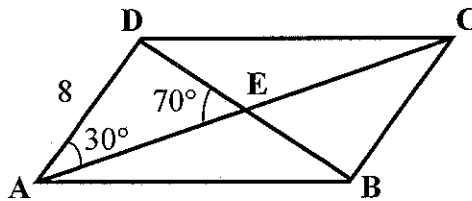


2012 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Let $i = \sqrt{-1}$. Then $i^7 - i^{13} = ki$, where k is a real number. Find the value of k .
2. Find the sum of the first thirty terms of the arithmetic sequence: 1.23, 2.37, 3.51, ... Express your answer as a decimal.
3. In the diagram, $ABCD$ is a parallelogram in which the diagonals intersect at E . If $\overline{AD} = 8$, $\angle DAC = 30^\circ$ and $\angle DEA = 70^\circ$, find the length of diagonal \overline{BD} . Express your answer as a **decimal** rounded to the nearest hundredth.



4. A right circular cylinder has a **total** surface area of 132π . Find the maximum volume of such a right circular cylinder. Express your answer as a **decimal** rounded to the nearest hundredth.
5. If the magnitude of the three dimensional vector $(2, 3, p)$ is $\sqrt{38}$, find the smallest possible value of p .
6. Let x be an integer such that $0 < x < 150$. Find the sum of all possible distinct values of x such that $\cos(2x + 8)^\circ > 0$ and $\sin(5x - 12)^\circ < 0$.
7. A trapezoid has sides with respective lengths: 2, 41, 20, 41. Find the length of an altitude of this trapezoid.
8. Rounded to the nearest centimeter, find the circumference of a circle in which a chord whose length is 80 centimeters is 9 centimeters from the center of the circle.
9. When two boys mow a lawn, one works at twice the rate of the other. If it takes the two boys a total of 3 hours to mow the lawn when they work together, how many hours would it take the faster boy to mow the lawn by himself?
10. A triangle has vertices at $(0,0)$, $(5,6)$, and $(5,-2)$. A point is selected at random in the interior of the triangle. Find the probability that the point selected lies in Quadrant I. Express your answer as a common fraction reduced to lowest terms.

11. Solve the determinant equation for k : $\begin{vmatrix} 1 & -2 & -6 \\ 4 & 1 & 0 \\ 5 & -3 & k \end{vmatrix} = 165$.
12. Let the equation of a parabola be $y = x^2 - kx + w$. The sum of the squares of the x -intercepts of the parabola is 4210. If 18 times the sum of the x -intercepts of the parabola is subtracted from the product of the x -intercepts of the parabola, the result is 183. Find the least possible value of y .
13. When $2.2\overline{3}$ (where only the 3 repeats) is written as an improper fraction reduced to lowest terms, what is the value?
14. Let $f(x) = 2x^2 + 3$ and let $g(x) = kx + w$ where k and w are positive integers. If $f(g(3)) = 5411$, find the largest possible value of $(k + w)$.
15. Let $P(x) = 0$ be a third degree polynomial equation with **integer** coefficients. Let the polynomial equation have at least one root that is an **integer**. If $P(5) = 6$, $P(4) = -2$, and $P(-2) = -92$, find $P(13)$.
16. If $(2x + 3y)^8$ is expanded and completely simplified, then kx^3y^5 would be one of the terms. Find the value of k .
17. $\begin{bmatrix} 7 & 3 & -2 \\ 8 & 6 & 12 \end{bmatrix} + \begin{bmatrix} 11 & -3 & 3 \\ 1 & 7 & -5 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ Find the value of $acd - 2cf$.
18. The first three terms of an arithmetic progression are respectively: $3x - 7$, $8x - 58$, and $2x + 78$. Find the value of x .
19. Find the vertex of the parabola whose equation is $y = x^2 + 4x + 11$. Express your answer as an **ordered pair** of the form (x, y) .
20. If $k(\log(9)) = \log(27)$, find the value of k . Express your answer as a **decimal**.

Name: _____ **ANSWERS** _____

Team Code: _____

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Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. -2

2. 532.8 **Must be this decimal**

3. 8.51 **Must be this decimal.**

4. 648.36 **Must be this decimal, cubic units optional**

5. -5

6. 2167

7. 40

8. 258 **Centimeters optional**

9. 4.5 **Hours optional**

10. 3/4 **Must be this reduced common fraction**

11. 7

12. -1429

13. 67/30 **Must be this reduced improper fraction**

14. 50

15. 1078

16. 108,864

17. 148

18. 17

19. (-2,7) **Must be this ordered pair**

20. 1.5 **Must be this decimal**