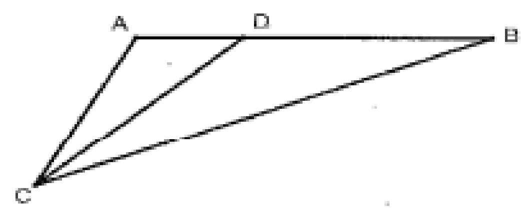
2015 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. Given $f(x) = 2x - 5$, for what value(s) of a does $f(a) = f(f(a))$?

2. If $\frac{5+\sqrt{3}}{3-\sqrt{3}} = \frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are integers with $d > 0$, find the minimum value of the product $abcd$.

3. In the diagram, points A, D, and B are collinear. $\angle BAC = 112°$, $\angle ACD = 18°$, and $\angle ABC = 34°$. Find the degree measure of $\angle DCB$.



4. When the polynomial $x^5 - 2x^3 + x + k$ is divided by $x - 2$, the remainder is 2. Find the value of k.

5. Find the value of k if the graph of $f(x) = \frac{3kx - 10}{4x + k}$ has an $x-$intercept of 30. Express your answer as a reduced common or improper fraction.

6. The tangent of one of the acute angles of a right triangle is 1. If one of the legs of this right triangle has a length of $7\sqrt{2}$, find the length of the hypotenuse of this right triangle.

7. Find the sum of all distinct values of y that satisfy the system $\begin{cases} \log_4(x) = \log_2(y) \\ x^2 - 9y^2 + 8 = 0 \end{cases}$. Put your answer in the form $a + b\sqrt{c}$.

8. Two numbers are selected at random in the interval $[0,5]$. Find the probability that the sum of the squares of these numbers is less than 5. Express your answer as a **decimal** rounded to the nearest **ten-thousandth**.

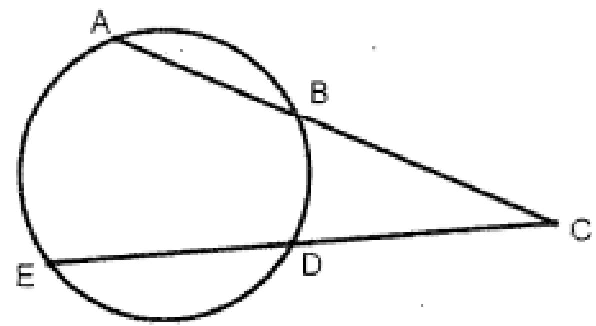
9. The vector $\langle 7.1, 8.2 \rangle$ is perpendicular to the vector $\langle -4.1, k \rangle$. If both vectors are in standard position, find the value of k. Express your answer as an **exact decimal**.

10. If the seven letters of the word *BANANAS* are written in random order, find the probability that the three A's are consecutive letters. Express your answer as a common fraction reduced to lowest terms.

11. One of the interior angles of a convex decagon measures $23°$. What is the maximum number of interior angles of that convex decagon that could be supplementary to the $23°$ angle?

12. A right triangle having legs of lengths $10\sqrt{3}$ and $24\sqrt{3}$ is inscribed in a circle of area $A\pi$. Find the value of A. Give your answer rounded to two decimal places.

13. In the diagram, points A, B, D, and E lie on the circle. Points A, B, and C are collinear and points C, D, and E are collinear. If $CD = 4.6$, $DE = 4.8$, and $BC = 3.2$, find the length of $\overline{AC}$. Express your answer as an **exact decimal**.



14. Find the value of k for which the graph of $y = 16x^2 + 2x + k$ is tangent to the $x-$axis. Express your answer as a reduced common or improper fraction.

15. The first term of a geometric sequence is 1, the second term is -2, and the third term is 4. Find the sum of the fifth and seventh terms.

16. The sum of the first four terms of an arithmetic sequence is 106, and the sum of the first five terms of this arithmetic sequence is 180. Find the second term of this sequence.

17. Find the **exact** length of the minor axis of the conic section whose equation is $x^2 - 2x + 3y^2 - 12y + 4 = 0$.

18. Let $i = \sqrt{-1}$. Find the value of $i - i^2 + i^3 + i^4 - i^5 + i^6 - i^7 + i^8$.

19. Let $f(x) = 27x^2 - 32x + 10$. Find the absolute value of the difference between the distinct values of x such that $f(x) = x$. Express your answer as a reduced common or improper fraction.

20. Assume that the probability that Jayden will make any free throw is 73%. Find the maximum whole number of free throws that Jayden can shoot such that his probability of making at least one free throw is less than 99%.

Name: ______________________ Team Code: _______

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Junior/Senior Indiyidual Test

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20. ____________________

Name: ______ANSWERS______ Team Code: _______

2015 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value (1 point).

1. **5**
2. **324**
3. **16** — Degrees optional
4. **−16**
5. $\frac{1}{9}$ — Must be this reduced fraction.
6. **14**
7. $1+2\sqrt{2}$ — Must be in this exact form.
8. **0.1571** — Must be in this exact form.
9. **3.55** — Must be in this exact form.
10. $\frac{1}{7}$ — Must be this reduced fraction.
11. **8**
12. **507** — Corrected in Grading Room
13. **13.5125** — Must be in this exact form.
14. $\frac{1}{16}$ — Must be this reduced fraction.
15. **80**
16. **17**
17. $2\sqrt{3}$ — Must be in this exact form.
18. **2**
19. $\frac{1}{9}$ — Must be this reduced fraction.
20. **3**

Awards Lists and Solutions to the Team Competition may be found at http://math.nku.edu/job